Spin Caloritronics III, Lorentz Center, Leiden 2011

SPIN CURRENT FROM MECHANICAL MOTION

Jun'ichi Ieda

Mamoru Matsuo, Eiji Saitoh, & Sadamichi Maekawa

Advanced Science Research Center, Japan Atomic Energy Agency

Ref. Phys. Rev. Lett. 106, 076601(2011) & arXiv:1104.3644v1.

SPIN-MECHATRONICS?

The 1st law of T.D. $$\Delta U = \Delta W + Q$$

CONTENTS

Motivation

Fundamental theory of the coupling between mechanical rotation & spin current?

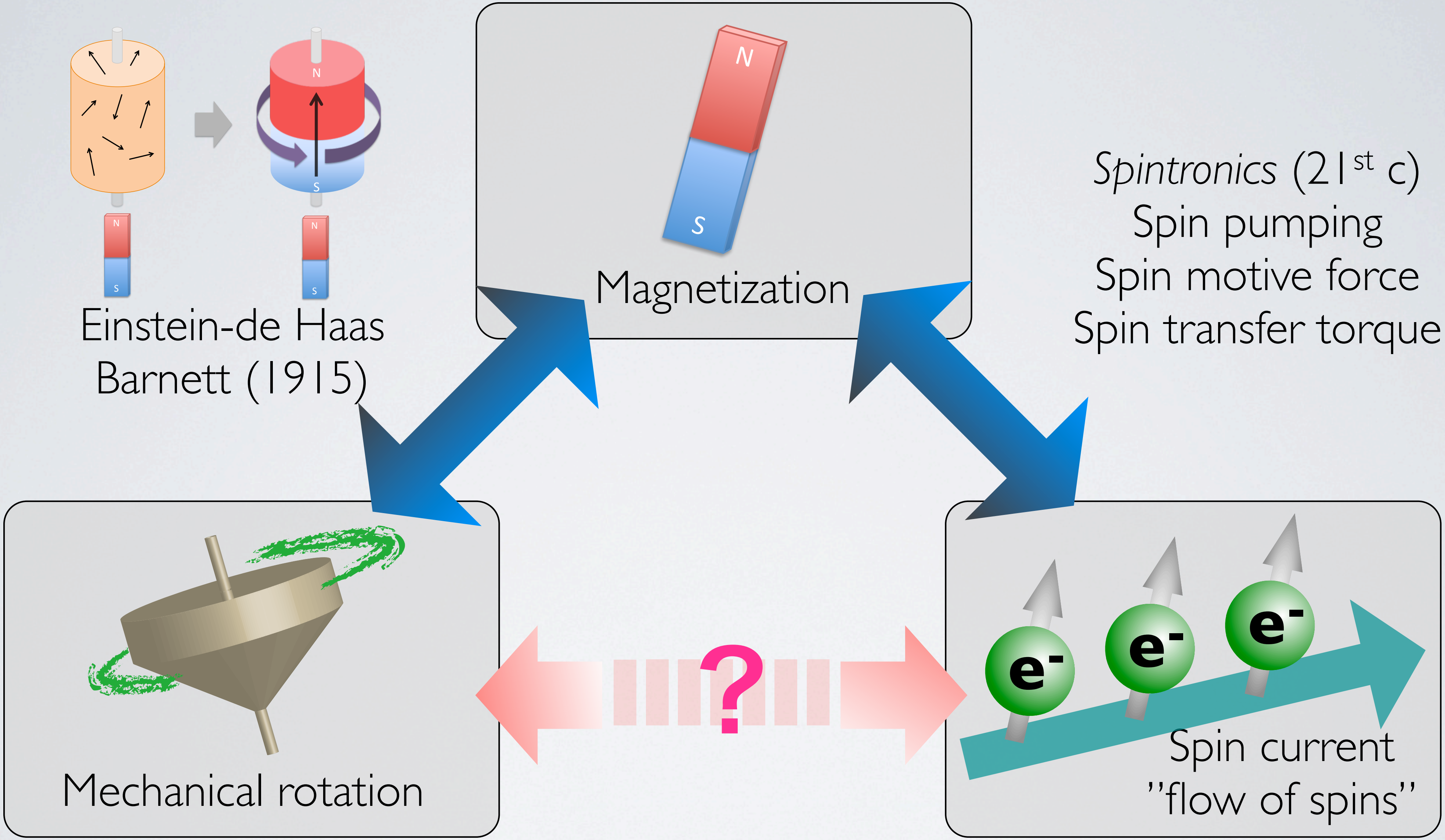
Backgroud

- Einstein's discoveries in 1915
- Spin Hall Effect - "special relativistic effect in condensed matter"

Formalism & result

- Low energy limit of general relativistic Dirac equation
- Spin current generation from rotating bodies
  - "general relativistic effect in condensed matter"

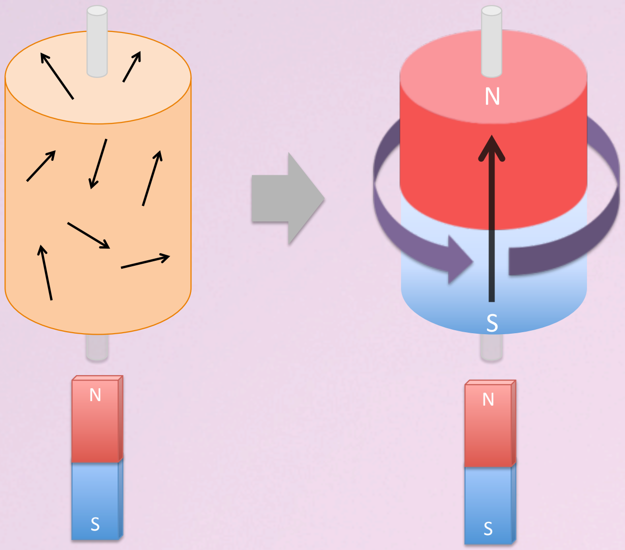
MOTIVATION



Fundamental theory of the direct coupling
between mechanical rotation & spin current?

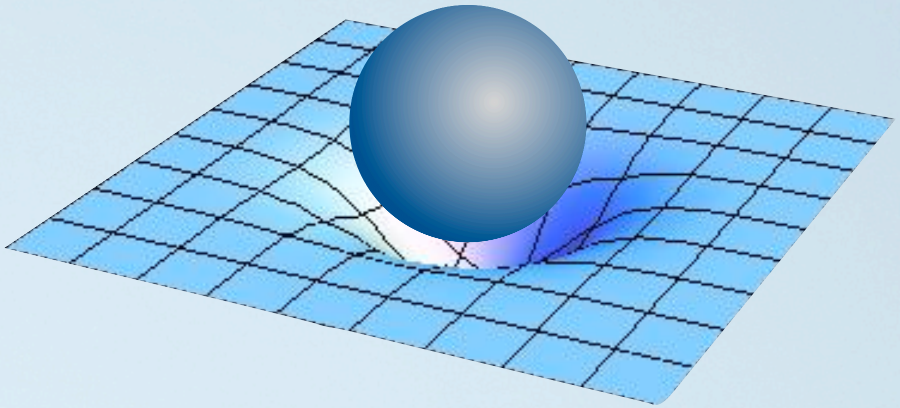
EINSTEIN'S DISCOVERIES IN 1915

Einstein-de Haas effect



Angular momentum conservation
of magnetization & rotation

General Relativity



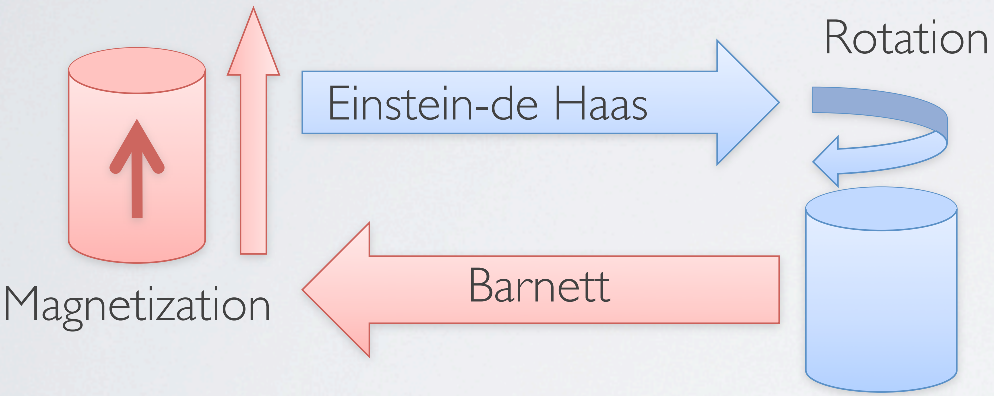
Theory of gravitation & inertial forces

MAGNETIZATION & ROTATION

In 1915

Einstein-de Haas effect

Barnett effect



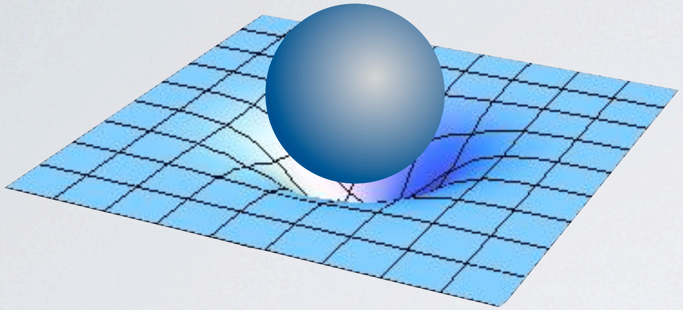
Deflection angle

→ Gyromagnetic ratio of electron

"Einstein's Only Experiment"

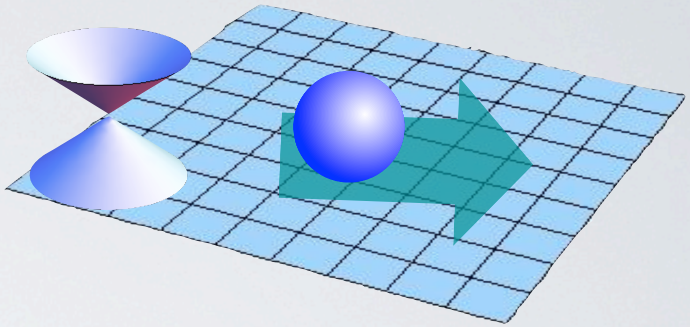
GENERAL RELATIVITY: RECIPE FOR INERTIAL EFFECTS

General Relativity (1915)



Theory of gravitation &
inertial effects in accelerating frames

Special Relativity (1905)



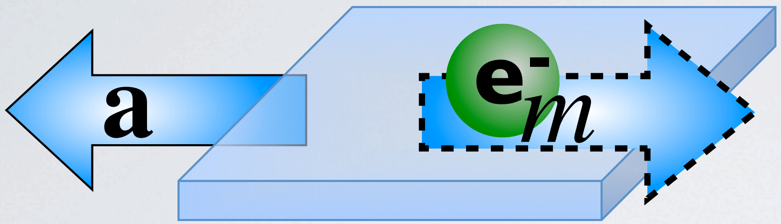
Theory in inertial frame

Einstein's equivalence principle
"Equivalence of gravity and inertial force"

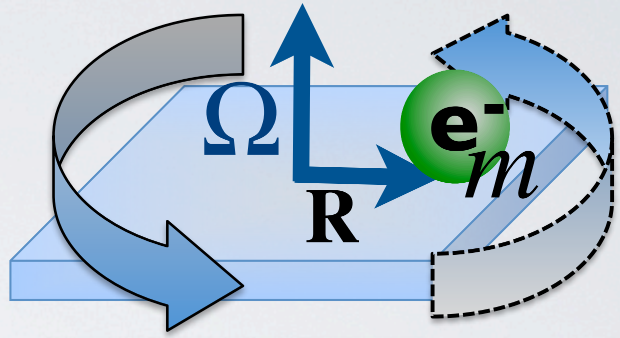
Recipe for inertial forces in accelerating frames

INERTIAL FORCES

Linear acceleration



Rotational acceleration



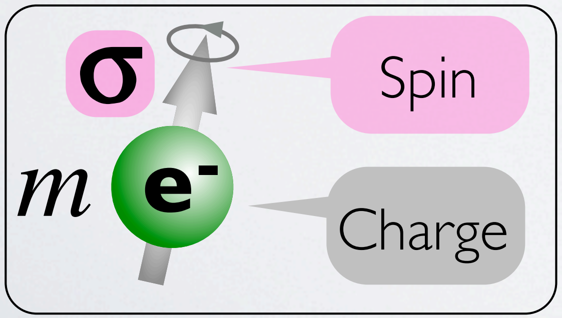
$$-m\mathbf{a} = -e\boxed{(e/m)^{-1}\,\mathbf{a}}$$

$\mathbf{E}_{\mathrm{eff}}$

$$-m\mathbf{v}\times\boldsymbol{\Omega} = -e\mathbf{v}\times\boxed{(e/m)^{-1}\,\boldsymbol{\Omega}}$$

Coriolis

$\mathbf{B}_{\mathrm{eff}}$



Inertial effects on spins?

Consult **general relativity** about **inertial effects on spins!**

DIRAC EQUATIONS

E-M and gravitational fields can be included by introducing "covariant derivatives."

Dirac eq. in vacuum

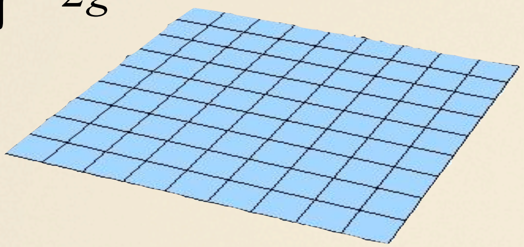
$$\left[\gamma^{\mu}\partial_{\mu}+\frac{mc}{\hbar}\right]\psi=0$$

$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2g^{\mu\nu}$$

Dirac eq. with E-M fields

$$\left[\gamma^{\mu}\left(\partial_{\mu}-i\frac{q}{\hbar}A_{\mu}\right)+\frac{mc}{\hbar}\right]\psi=0$$

$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2g^{\mu\nu}$$

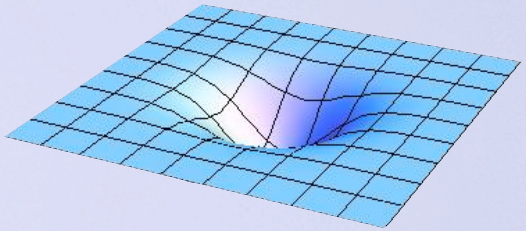


Flat space-time (inertial frame)

Dirac eq. with E-M & gravitational fields

$$\left[\gamma^{\mu}(x)\left(\partial_{\mu}-\Gamma_{\mu}\left[g(x)\right]-i\frac{q}{\hbar}A_{\mu}\right)+\frac{mc}{\hbar}\right]\psi=0$$

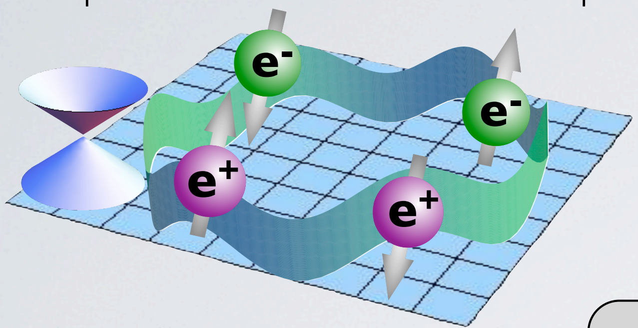
$$\left\{\gamma^{\mu}(x),\gamma^{\nu}(x)\right\}=2g^{\mu\nu}(x)$$



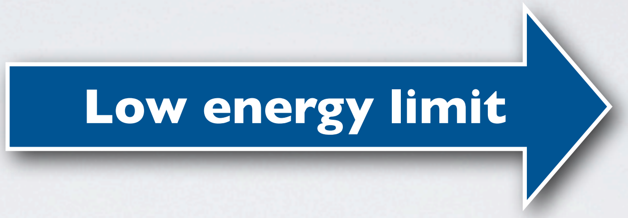
Curved space-time (noninertial frame)

QM IN INERTIAL /NONINERTIAL FRAME

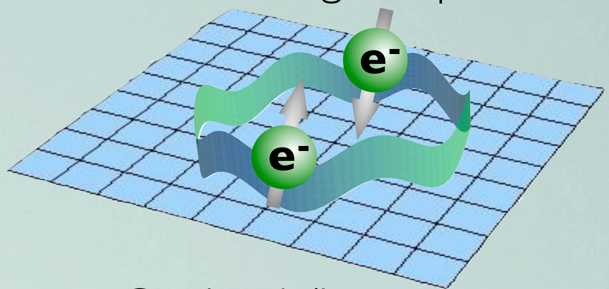
Special Relativistic Dirac eq.



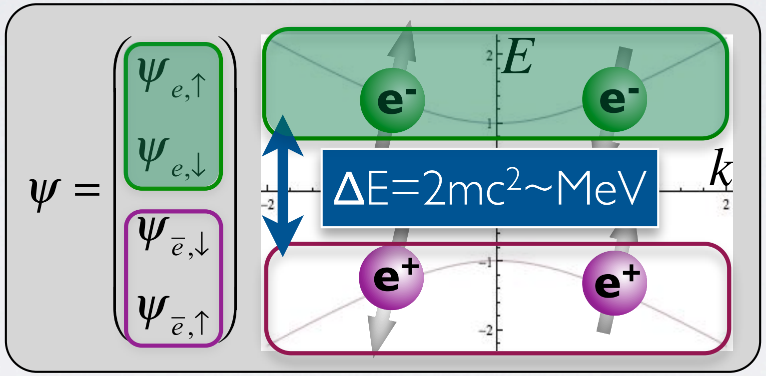
Spin-1/2 particle in inertial frame



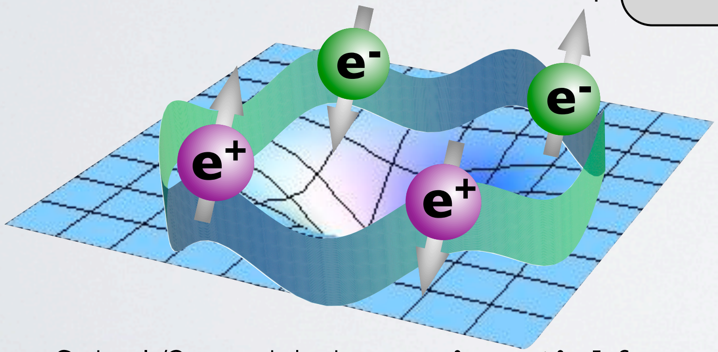
Pauli-Schrödinger equation



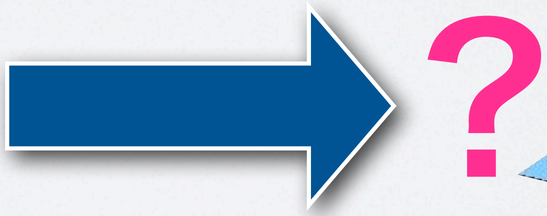
- Coulomb/Lorentz
- Zeeman
- Spin-Orbit



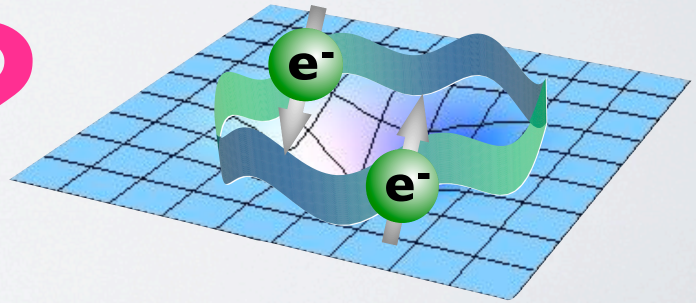
General Relativisitc Dirac eq.



Spin-1/2 particle in **noninertial** frame



Pauli-Schrödinger equation
in **noninertial** frame

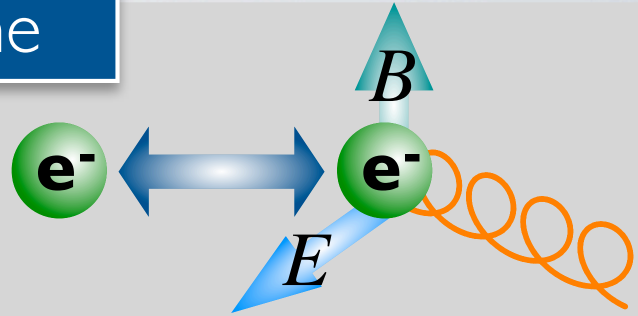


PAULI-SCHRÖDINGER EQ. IN INERTIAL FRAME

Low energy limit of Dirac eq. in inertial frame

$$H = \frac{1}{2m}(\mathbf{p} + e\mathbf{A})^2 + e\phi$$

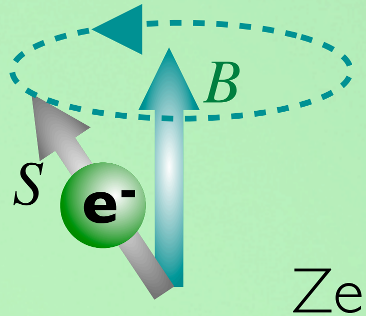
Coulomb
& Lorentz



Coulomb & Lorentz

$$-\mu_B \boldsymbol{\sigma} \cdot \mathbf{B}$$

Zeeman

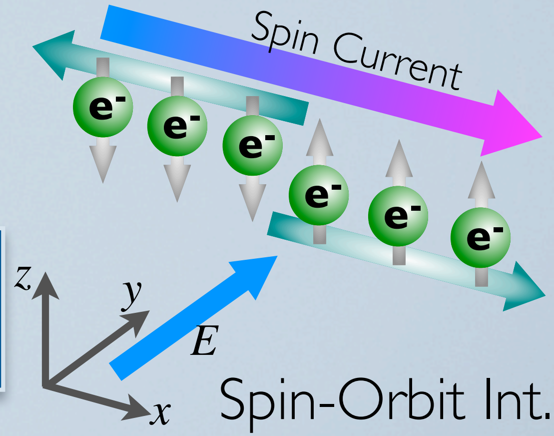


Zeeman

$$+\frac{\lambda}{\hbar}\boldsymbol{\sigma} \cdot \left[(\mathbf{p} + e\mathbf{A}) \times (-e)\mathbf{E}\right]$$

Spin-Orbit Int.

"Special relativistic effect"
originating from Dirac eq.



Spin-Orbit Int.

ENHANCEMENT OF SOI IN COND. MATTER

Spin-Orbit Interaction $\frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p}+e\mathbf{A})\times(-e)\mathbf{E}\right]$

In Vacuum $$\frac{\lambda(mv)^2}{\hbar^2}=\left(\frac{v_e}{c}\right)^2\sim 10^{-4}$$

Enhancement of SOI in matter

In Pt $$\frac{\lambda(mv)^2}{\hbar^2}=\lambda k_F^2\sim 0.58$$

Discoveries of large SOI systems

- Pt, ...
- GaAs, InAs, ...

Vila, Kimura, and Otani,
Phys. Rev. Lett. 88, 226604 (2007)

SPIN HALL EFFECT IN ROTATING FRAME

$$H = \frac{1}{2m}(\mathbf{p}+e\mathbf{A})^2 + e\phi - \mu\boldsymbol{\sigma}\cdot\mathbf{B} + \frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p}+e\mathbf{A})\times(-e)\mathbf{E}\right]$$

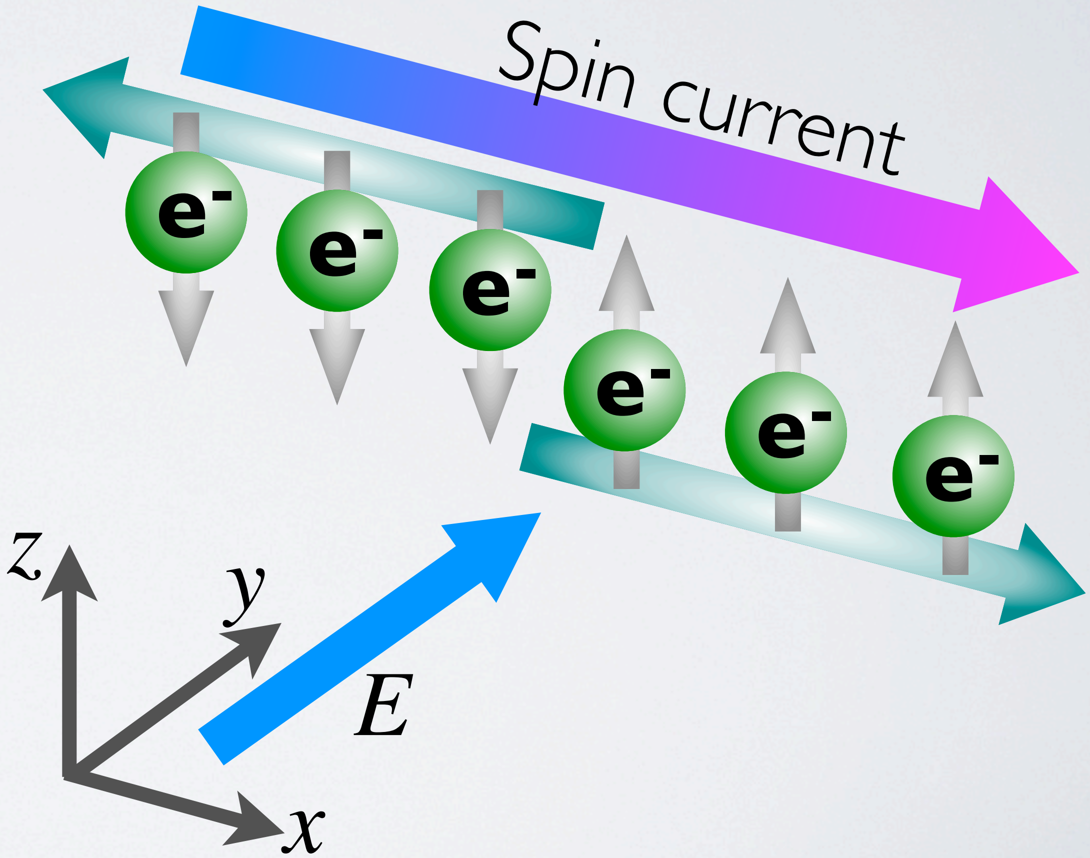
Spin-Orbit Interaction

$$\frac{d}{dt}\mathbf{r} = \mathbf{v} - \frac{e\lambda}{\hbar}\boldsymbol{\sigma}\times\mathbf{E}$$

$\pm x$ $\pm z$ y

Spin-dependent velocity

Spin current generated perpendicular to E-field

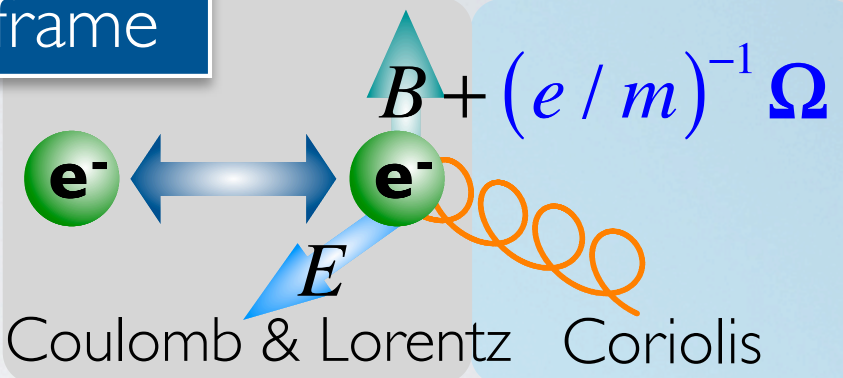


PAULI-SCHRÖDINGER EQ. IN ROTATING FRAME

Low energy limit of Dirac eq. in rotating frame

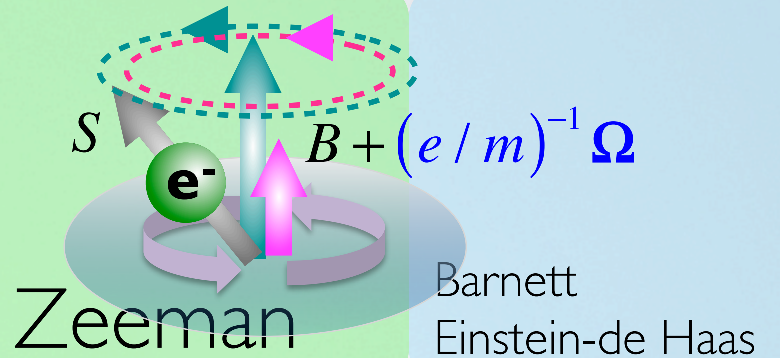
$$H = \frac{1}{2m}(\mathbf{p} + e\mathbf{A})^2 + e\phi \quad -\mathbf{L}\cdot\boldsymbol{\Omega}$$

Coulomb & Lorentz | Coriolis



$$-\mu_B \boldsymbol{\sigma}\cdot\left(\mathbf{B} + (e/m)^{-1}\boldsymbol{\Omega}\right)$$

Zeeman | Barnett Einstein-de Haas



$$+\frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p} + e\mathbf{A})\times(-e)\left(\mathbf{E} + (\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}\right)\right]$$

NEW!

Gravitational-Spin-Orbit

"General relativistic effect in condensed matter"

G-Spin-Orbit

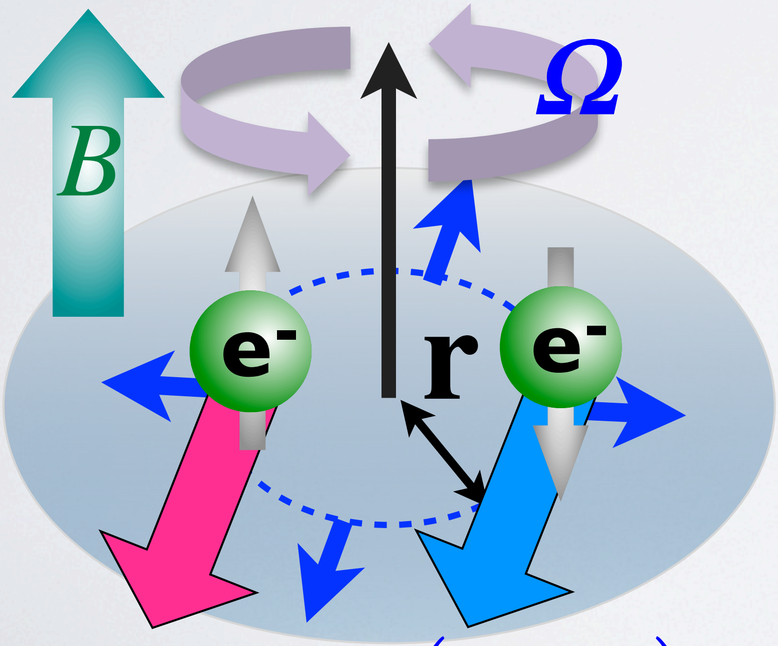
ANOMALOUS VELOCITY IN ROTATING FRAME

$$+\frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p}+e\mathbf{A})\times(-e)\left(\mathbf{E}+(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}\right)\right]$$

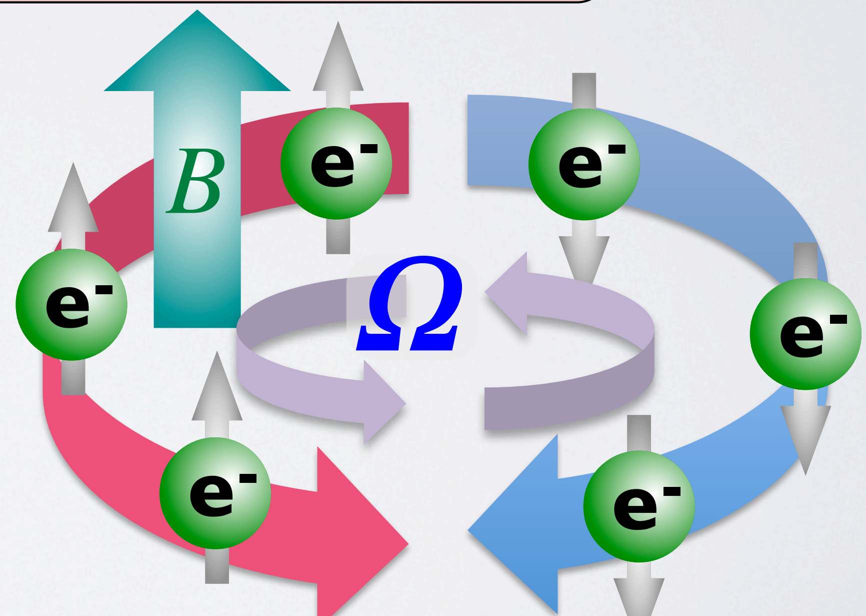
NEW!

$$\frac{d}{dt}\mathbf{r}=\mathbf{v}-\frac{e\lambda}{\hbar}\boldsymbol{\sigma}\times\left(\mathbf{E}+(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}\right)$$

±Azimuthal | $\pm z$ | $\mathbf{E}_{rot}$ | **Radial**



$$\mathbf{E}_{rot}=(\Omega\cdot B)\,\mathbf{r}$$



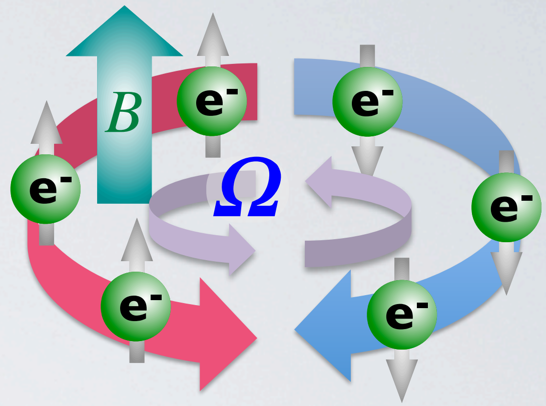
Spin current generated
in azimuthal direction

SPIN CURRENT FROM MECHANICAL ROTATION

$$+\frac{\lambda}{\hbar}\boldsymbol{\sigma}\cdot\left[(\mathbf{p}+e\mathbf{A})\times q\left(\mathbf{E}+(\boldsymbol{\Omega}\times\mathbf{r})\times\mathbf{B}\right)\right]$$

NEW!

Gravitational-Spin-Orbit Int.

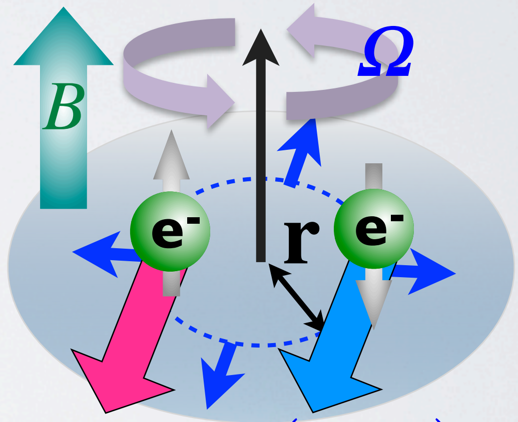


New spin Hall effect - "general relativistic effect in matter"

$$J_s^{\Omega+B}(r)=2ne\lambda k_F^2\frac{\hbar\Omega}{\varepsilon_F}\times\frac{eB}{m^*}\times r$$

Pt (Ballistic case)
r = 0.1m
B = 1T, Omega = 1kHz

$\rightarrow J_s = 10^8 A/m^2$



$$\mathbf{E}_{rot}=(\Omega\cdot B)\mathbf{r}$$

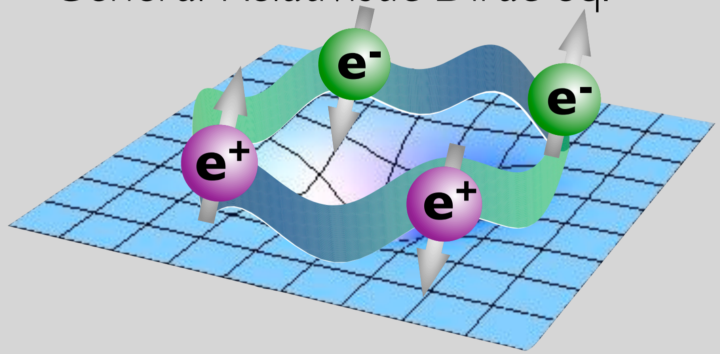
Ref. M. Matsuo, J. Ieda, E. Saitoh, and S. Maekawa, Phys. Rev. Lett. 106, 076601(2011)

WHY OVERLOOKED ?

A. Output has been missing ⇒ Spin Current!

"Spin-Rotation Coupling"
de Oliveira-Tiomno, Nuovo Cimento 1962
Mashhoon, Phys. Rev. Lett. 1988
Hehl-Ni, Phys. Rev. D 1990

General Relativistic Dirac eq.

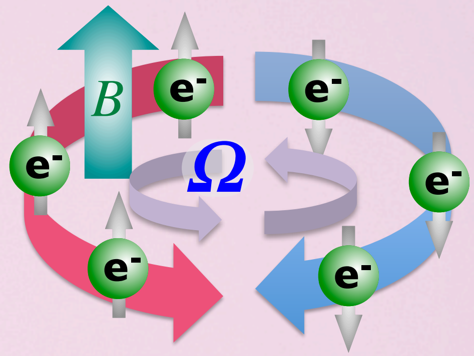


Theory of Spin-1/2 particle in inertial frames

Coupling of rotation with **Spin**

Detection technique: ISHE

Enhancement of ISO in Cond. Matter



NEW!

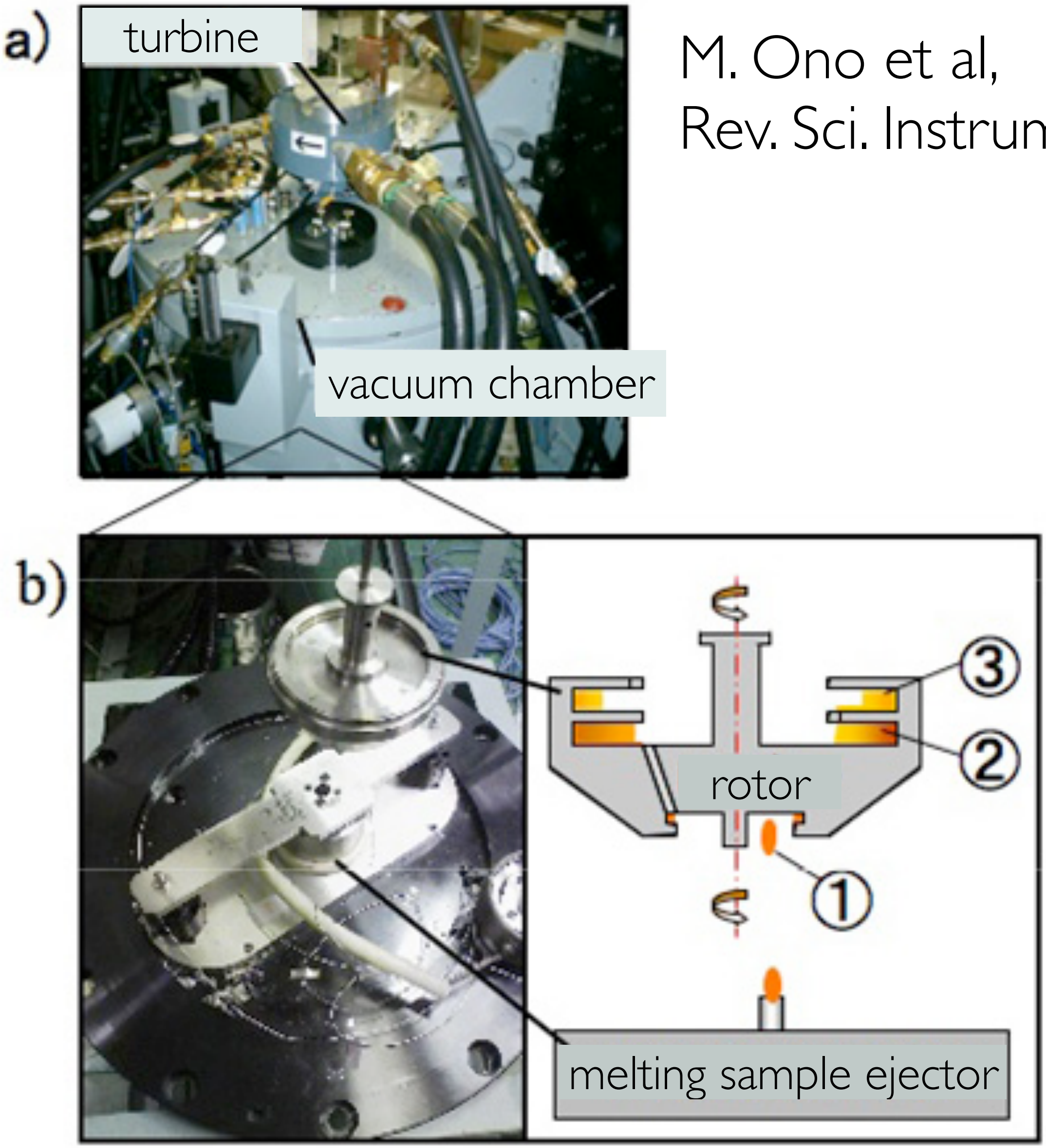
Coupling of rotation with **Spin Current**

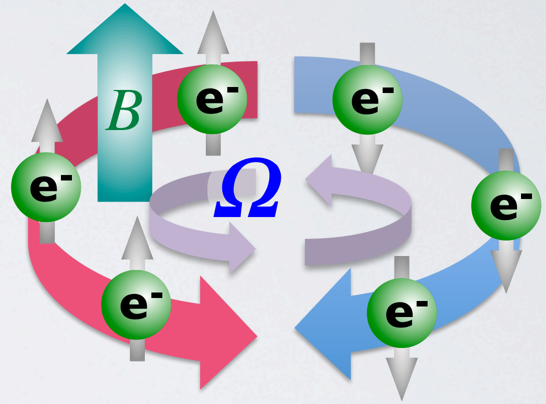
Spin current form rotation

"gen. rel. effect" in cond. matter

ULTRA HIGH SPEED ROTOR IN JAEA

M. Ono et al,
Rev. Sci. Instrum. 80, 082908 (2009)





On going experiment for generation/detection of spin current due to mechanical rotation!

IMPURITY SCATTERING

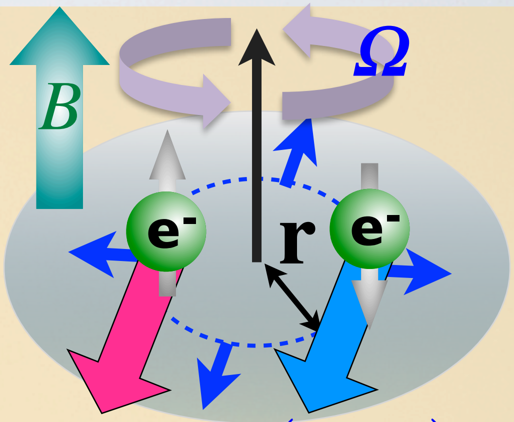
$$\dot{\mathbf{r}} \cdot \frac{\partial f_{\boldsymbol{\sigma}}}{\partial \mathbf{r}} + \dot{\mathbf{k}} \cdot \frac{\partial f_{\boldsymbol{\sigma}}}{\partial \mathbf{k}} = -\frac{f_{\boldsymbol{\sigma}} - f_0}{\tau},$$

$$f_{\boldsymbol{\sigma}} = f_0 + e\mathbf{v} \cdot \tau \frac{\mathbf{E}'_\sigma + \tau \boldsymbol{\omega}_c \times \mathbf{E}'_\sigma}{1 + (\tau\omega_c)^2} \frac{\partial f_0}{\partial \varepsilon}$$

$$\mathbf{J}_s = J_s^r \hat{\mathbf{e}}_r + J_s^\phi \hat{\mathbf{e}}_\phi$$

radial (longitudinal) spin current: $J_s^r = \frac{(\omega_c \tau)}{1 + (\omega_c \tau)^2} J_s$

azimuthal (transverse) spin current: $J_s^\phi = \frac{(\omega_c \tau)^2}{1 + (\omega_c \tau)^2} J_s$



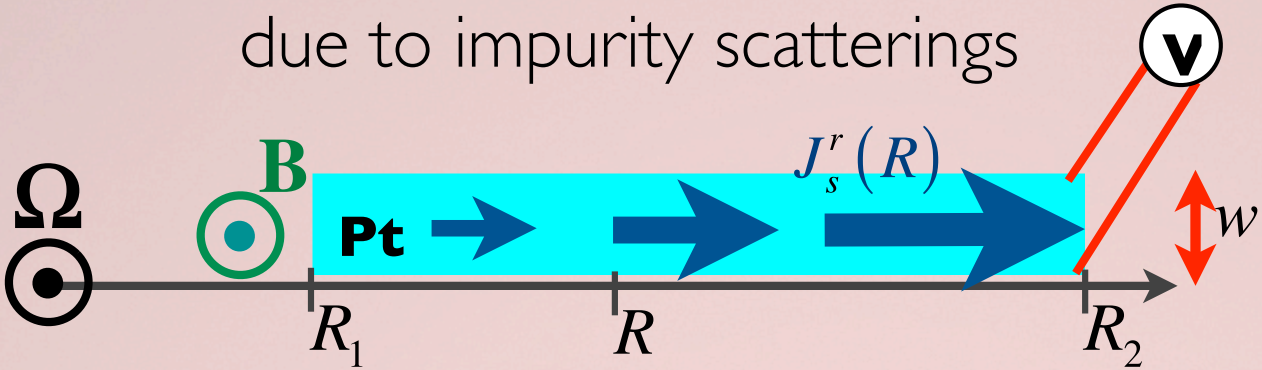
$$\mathbf{E}_{rot} = (\Omega \cdot B)\mathbf{r}$$

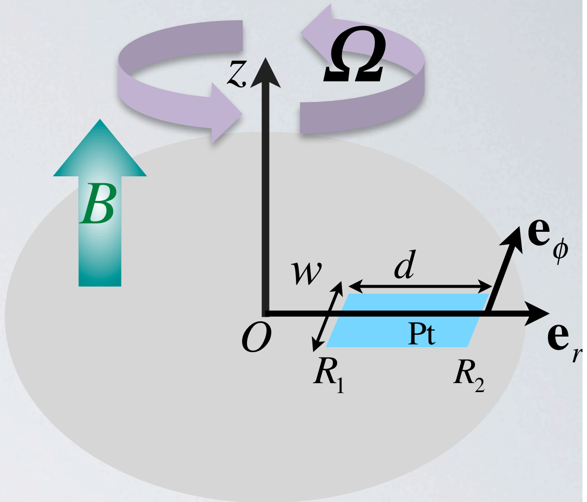
$$J_s = 2ne\lambda k_F^2 \frac{\hbar\Omega}{\varepsilon_F} \frac{eB}{m^*} R$$

Ref. M. Matsuo, J. Ieda, E. Saitoh, and S. Maekawa, to appear in Appl. Phys. Lett.

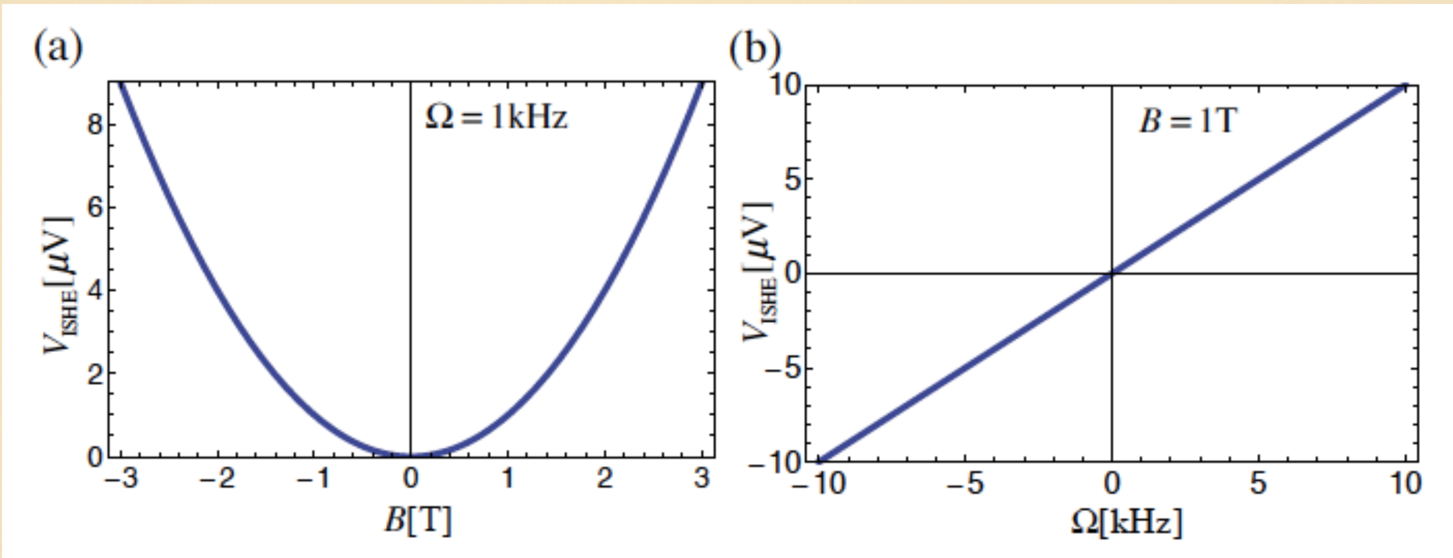
DETECTION METHOD

Spin current is generated in radial direction
due to impurity scatterings



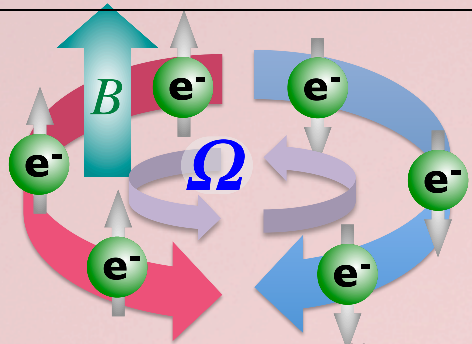


Inverse spin Hall voltage: $V_{ISHE} = \theta w \rho_N \langle J_s^r \rangle$



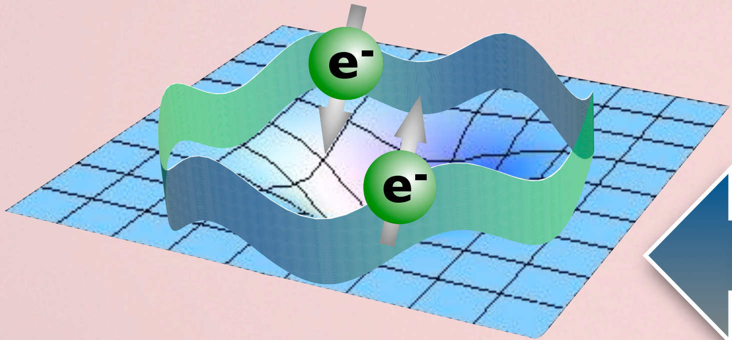
SUMMARY

Spin current generated from mechanical rotation



"General relativistic effect in matter"

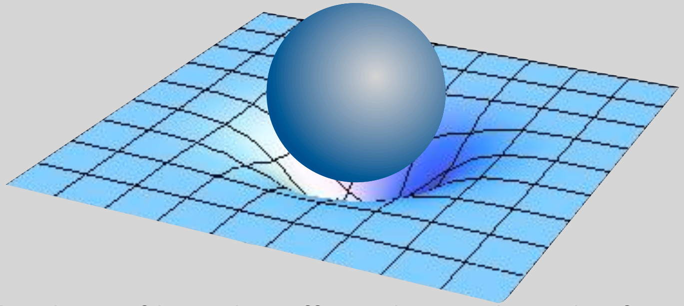
Renormalizaion of SOI



New Gravitational-Spin-Orbit Interaction

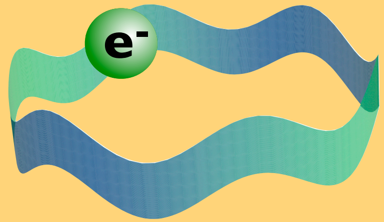
Pauli-Schrödinger equation in rotating frame

General Relativity

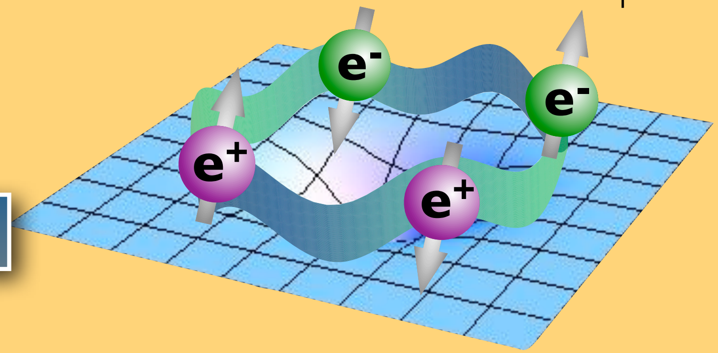


Recipe of inertial effects in noninertial frame

Quamtum Mechanics



General Relativisitc Dirac eq.



Spin-1/2 particle in noninertial frame

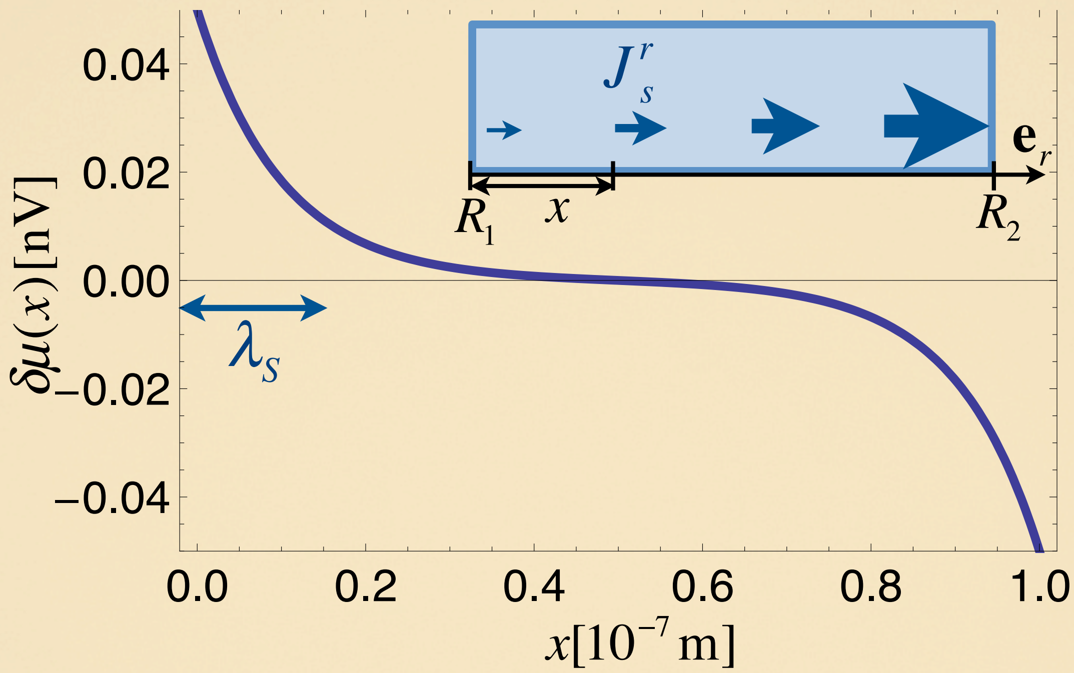
Low energy limit

BACKUP SLIDES

SPIN ACCUMULATION

Spin diffusion equation in the radial direction:

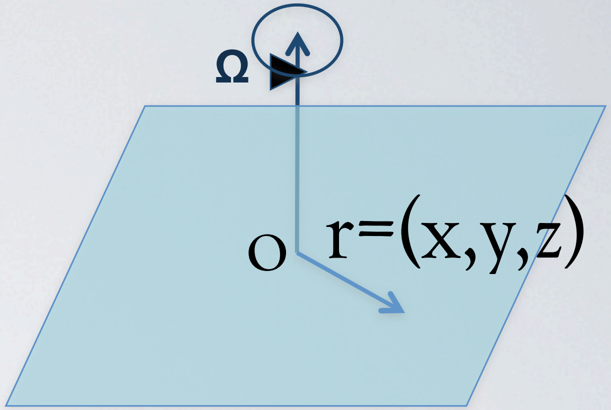
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\delta\mu}{\partial r}\right)=\frac{\delta\mu}{\lambda_s^2}-\frac{e\rho}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{2ne\kappa\tau\omega_c^2 r}{1+(\tau\omega_c)^2}\right)$$



Ref. M. Matsuo, J. Ieda, E. Saitoh, and S. Maekawa, to appear in Appl. Phys. Lett.

METRIC OF UNIFORMLY ROTATING FRAME

$$\frac{d}{dt}[\bullet] \Rightarrow \frac{d}{dt}[\bullet] - \boldsymbol{\Omega} \times [\bullet]$$



$$d\mathbf{r}' = d\mathbf{r} + (\boldsymbol{\Omega} \times \mathbf{r})\,dt$$

$$ds^2 = \left(-c^2 + (\Omega \times \mathbf{r})^2\right) dt^2 + 2(\Omega \times \mathbf{r})\, d\mathbf{r}\, dt + d\mathbf{r} \cdot d\mathbf{r}$$

$$= (cdt, dx, dy, dz) \begin{pmatrix} -1 + (\Omega \times \mathbf{r})^2 & (\Omega \times \mathbf{r})_1 & (\Omega \times \mathbf{r})_2 & (\Omega \times \mathbf{r})_3 \\ (\Omega \times \mathbf{r})_1 & 1 & 0 & 0 \\ (\Omega \times \mathbf{r})_2 & 0 & 1 & 0 \\ (\Omega \times \mathbf{r})_3 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

Space-time dependent metric

= curved space-time

GENERAL COORDINATE TRANSFORMATION

$$\begin{pmatrix} dt' \\ dx' \\ dy' \\ dz' \end{pmatrix} = \begin{pmatrix} \partial t'/\partial t & \partial t'/\partial x & \partial t'/\partial y & \partial t'/\partial z \\ \partial x'/\partial t & \partial x'/\partial x & \partial x'/\partial y & \partial x'/\partial z \\ \partial y'/\partial t & \partial y'/\partial x & \partial y'/\partial y & \partial y'/\partial z \\ \partial z'/\partial t & \partial z'/\partial x & \partial z'/\partial y & \partial z'/\partial z \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Omega t & -\sin\Omega t & 0 \\ 0 & \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} dt' \\ dx' \\ dy' \\ dz' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\Omega y & \cos\Omega t & -\sin\Omega t & 0 \\ \Omega x & \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} \qquad R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\Omega y & \cos\Omega t & -\sin\Omega t & 0 \\ \Omega x & \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

GENERAL COORDINATE TRANSFORMATION

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\Omega y & \cos\Omega t & -\sin\Omega t & 0 \\ \Omega x & \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F' = R^T F R$$

$$\Rightarrow \begin{cases} E' \approx E + (\Omega \times r) \times B \\ B' \approx B \end{cases}$$