Domain wall effects in ferromagnet-superconductor structures

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We investigate how domain structures of the ferromagnet in superconductor-ferromagnet heterostructures may change their transport properties. We calculate the distribution of current in the superconductor induced by magnetic field of Bloch domain walls, e.g., find the “lower critical” magnetization of the ferromagnet that provides vortices in the superconductor.

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Superconductivity and ferromagnetism are two competing phenomena, while the first prefers antiparallel spin orientation of electrons in Cooper pairs, the second forces the spins to be aligned parallel. Their coexistence in one and the same material or their interaction in spatially separated materials leads to a number of new interesting phenomena, for example, π state of superconductor (S), ferromagnet (F), superconductor (SFS) Josephson junctions,[1–8] highly nonmonotonic dependence of the critical temperature T_c of a SF bilayer as a function of the ferromagnet thickness,[9] etc. Recent investigations of SF bilayers showed that their transport properties often strongly depend on the interplay between magnetic structure of the ferromagnet and superconductivity.[10–20] In particular, it was argued that due to ferromagnetic domains vortices may appear in the superconducting film of the SF bilayer and the domain configuration, in turn, may depend on the vortices.[13,14] Recently, a generation of vortices by magnetic texture of the ferromagnet in SF junctions was demonstrated experimentally.[20] In a number of experiments dealing with T_c of SF bilayers, or the Josephson effect in SFS structures, the domain magnetizations were parallel to the SF interface.[1] Ferromagnets used in the experiments were often dilute with the exchange field comparable to the superconducting gap and with small domain size, smaller or comparable to the bulk superconductor screening length, and broad domain walls.[1]

In this paper we try to answer the question of how domain structures of the ferromagnet in SF bilayers may change their transport properties. In the major part of the paper we discuss the junctions where S and F are weakly coupled, i.e., there is an insulator layer in-between such that there is no proximity effect. We assume that magnetizations of the domains are parallel to the SF interface. We find the distribution of current in the S film induced by the magnetic field of the domain walls and the “lower critical” magnetization of the ferromagnet for which vortices become to proliferate into the S film. In the end of the paper we estimate the critical temperature of the superconducting transition in strongly coupled SF bilayers when the proximity effect is strong.

In this paper we do not consider the rearrangement of the domain configuration due to the superconductor.[13,14] We mention only that the superconductor can induce transitions between Bloch and Néel domain wall types. The point is that the crystal structure of the ferromagnets used in the experiments of Ref. 1 was not perfect. Experimental data suggest that defects, dislocations in the lattice that appear during the lithography process, stick to domain configurations. Also, we do not discuss the effect of the indirect Ruderman-Kittel-Kasuya-Yoshida (RKKY) interaction on the magnetic structure in the F film. As it is well known,[21] the indirect RKKY interaction is suppressed on the lengthscale larger than the coherence length ξ in the S film. Therefore, it can be neglected for the case of relatively thick S and F films where thicknesses d_S and d_F, correspondingly, are larger than the coherence length, $d_S, d_F \gtrsim \xi$.

The domain texture in the F film is described by the following magnetization (see Fig. 1):

$$\mathbf{M}(x,z) = M\,\theta(z)\,\theta(d_F - z)\sum_{j=-\infty}^{\infty}(-1)^j\mathbf{m}(x - jL), \qquad (1)$$

where M denotes the absolute value of the magnetization $\mathbf{M}$, L the width of a domain, and $\theta(z)$ the Heaviside step function. The unit vector $\mathbf{m}(x)$ rotates as follows:[22]

$$m_x = 0, \quad m_y = \tanh(x/\delta), \quad m_z = -1/\cosh(x/\delta), \qquad (2)$$

where δ stands for width of a domain wall. For reasons to be explained shortly, we consider the case of thin domain walls, $\delta \ll L$.

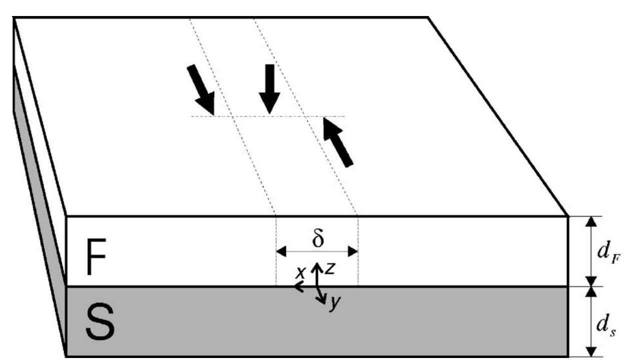

FIG. 1. The SF junction. A sketch of a Bloch domain wall. The magnetization rotates according to Eq. (2). Magnetization in the center of the domain wall is perpendicular to the S film.

The vector potential $\mathbf{A}$ satisfies the Maxwell-Londons equation

$$\nabla \times (\nabla \times \mathbf{A}) + \lambda_L^{-2}\theta(-z)\theta(z+d_S)\mathbf{A} = 4\pi \nabla \times \mathbf{M}. \quad (3)$$

Equation (3) should be supplemented by the standard conditions of continuity for $\mathbf{A}$ and $\partial\mathbf{A}/\partial z$ at the interfaces.[22] By solving Eq. (3) with the help of the Fourier transformation we can find the distribution of the magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ in the entire space.[23] In general, the widths δ and L ought to be determined self-consistently from consideration of the free energy that involves the exchange energy, energy of the anisotropy (it contains a part of dipole-dipole interaction energy), and the energy of the magnetic field $\mathbf{B}$.[24] The results obtained will be reported elsewhere.[23] Hereafter, we assume that δ and L are true self-consistently determined parameters.

In agreement with general expectations the z component of the magnetization in the F film that collects at domain walls results in the current flow in the S film. It is convenient to define the current averaged over the thickness of the S film, $J_y(x)=-(c/4\pi)\lambda_L^{-2}\int_{-d_S}^{0}A_y dz$, where c denotes the speed of light and λ_L the London penetration length for a bulk superconductor. Then we obtain[23]

$$J_y(x) = -2\pi cM\frac{\delta}{\lambda_L^2 L}\sum_{n=0}^{\infty}\frac{\sin q_n x}{Q_n\cosh\left(\frac{\pi}{2}q_n\delta\right)}(1-e^{-q_n d_F}) \times \frac{Q_n + q_n\tanh(Q_n d_S/2)}{Q_n^2+q_n^2+2Q_n q_n\coth(Q_n d_S)}, \quad (4)$$

where $q_n=\pi(2n+1)/L$ and $Q_n=\sqrt{q_n^2+\lambda_L^{-2}}$ describes the screening of the magnetic field in the S film. Equation (4) constitutes the principal result of the present paper. It allows us to compute the distribution of the current flow in the S film for a general set of parameters d_S, d_F, δ, L, and λ_L. Below we shall analyze two of the most interesting cases of thick $(d_S,d_F \gg \lambda_L)$ and thin $(d_S,d_F \ll \lambda_L)$ SF bilayers.

Thick SF bilayer. Equation (4) can be drastically simplified provided $d_S,d_F\gg\lambda_L$. The current $J_y(x)$ becomes independent on the thicknesses d_S and d_F of the S and F films and is given as

$$J_y(x) = -cM\frac{2\pi\delta}{\lambda_L^2 L}\sum_{n=0}^{\infty}\frac{\sin q_n x}{\cosh\frac{\pi}{2}q_n\delta}\frac{1}{Q_n(Q_n+q_n)}. \quad (5)$$

In order to understand the distribution of the current $J_y(x)$ as determined by Eq. (5), we shall first analyze the case of a *single* domain wall. Taking the limit $L\to\infty$ in Eq. (5), we obtain the following result for the current in the presence of a single domain wall in the F film:

$$\frac{J_y(x)}{cM} = -\frac{\delta}{\lambda_L}\int_0^{\infty}\frac{d\omega}{\sqrt{1+\omega^2}}\frac{\sin\frac{x}{\lambda_L}\omega}{\cosh\frac{\pi\omega\delta}{2\lambda_L}}\frac{1}{\omega+\sqrt{1+\omega^2}}. \quad (6)$$

The distribution of the current $J_y(x)$ is governed by the single parameter $\pi\delta/(2\lambda_L)$ as it is shown in Fig. 2. If the width δ of

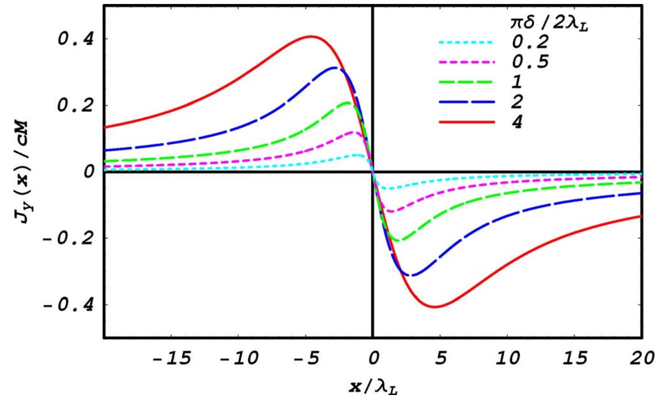

FIG. 2. (Color online) The plot of $J_y(x)/(cM)$ as a function of x/λ_L in the case of the thick SF bilayer, $d_S,d_F\gg\lambda_L$. The parameter $\pi\delta/(2\lambda_L)=0.2,0.5,1,2,4$ (from top to bottom in the left panel).

the domain wall is small compared to the London penetration length λ_L, $\pi\delta/(2\lambda_L)\ll 1$, we find the following distribution of the current in the S film:

$$J_y(x) = cM\begin{cases}\frac{\delta}{x}\left[\frac{|x|}{\lambda_L}K_1\left(\frac{|x|}{\lambda_L}\right)-1\right], & |x|\gg\delta,\\ \frac{x\delta}{2\lambda_L^2}\ln\frac{\pi\delta}{2\lambda_L}, & |x|\ll\delta,\end{cases} \quad (7)$$

where $K_1(x)$ is the modified Bessel function of the second kind. In the opposite case of the thick domain wall $\pi\delta/(2\lambda_L)\gg 1$ we obtain

$$\frac{J_y(x)}{cM} = \tanh\frac{x}{\delta} - \frac{2}{\pi}\,\mathrm{Im}\,\psi\left(\frac{1}{4}+i\frac{x}{2\pi\delta}\right) + \frac{\lambda_L}{\delta}\frac{\tanh\frac{x}{\delta}}{\cosh\frac{x}{\delta}}, \quad (8)$$

where $\psi(x)$ denotes the digamma function.

According to Eqs. (7) and (8) the current $J_y(x)$ behaves linearly with x for $x\ll\delta$ and decays as a power law for large x. The current distribution $J_y(x)$ is spread on the length $L_s \propto \max\{\delta,\lambda_L\}$ from the origin $x=0$. Its maximal absolute value $|J_y^m|\propto cM\delta/L_s = cM\min\{1,\delta/\lambda_L\}$.

Now we turn back to the general case of *multi* domain wall structures in the F film which correspond to a finite size L of domains. We have evaluated the sum in Eq. (5) numerically and present results for the current distribution in Fig. 3. While λ_L remains small compared with L the profile of $J_y(x)$ corresponds to almost independent current distributions near each domain wall that results in distinctive two maximum structure as shown in Fig. 3. When λ_L becomes of the order of L the two maximum structure transforms into sinusoidal-like profile with the maximum exactly in the middle of a domain.

Thin SF bilayer. In the case of the thin SF bilayer, $d_S,d_F\ll\lambda_L$, by expanding the general expression (4) in powers of d_S and d_F, we find

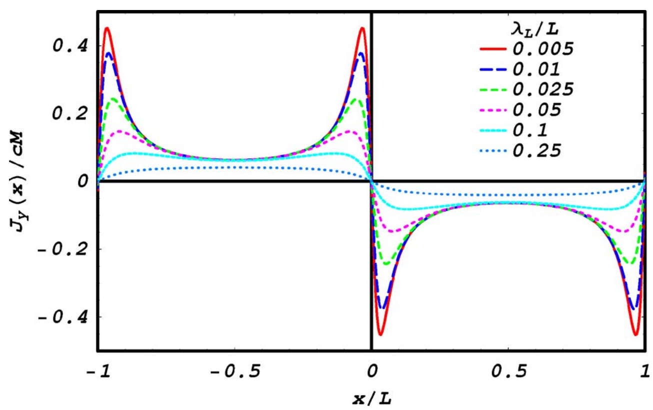

FIG. 3. (Color online) The plot of $J_y(x)/(cM)$ as a function of x/L in the case of the thick SF bilayer, $d_S, d_F \gg \lambda_L$. We use $\delta/L = 0.02$ and parameter $\lambda_L/L = 0.005, 0.01, 0.025, 0.05, 0.1, 0.25$ (from top to bottom in the left panel).

$$J_y(x) = -cMd_F \frac{2\pi\delta}{L}\sum_{n=0}^{\infty} \frac{\sin q_n x}{\cosh \frac{\pi}{2} q_n \delta} \frac{q_n}{1+2q_n\lambda}. \tag{9}$$

Here $\lambda = \lambda_L^2/d_S$ is usually referred to as the effective penetration length.[25] We shall first analyze the case of a *single* domain wall again. In the limit $L \to \infty$ we obtain from Eq. (9)

$$J_y(x) = -cM \frac{d_F \delta}{4\lambda^2}\int_0^{\infty} d\omega \frac{\omega}{1+\omega} \frac{\sin \frac{x}{2\lambda}\omega}{\cosh \frac{\pi\delta}{4\lambda}\omega}. \tag{10}$$

The distribution of the current $J_y(x)$ is presented in Fig. 4 for different values of the parameter $\pi\delta/(4\lambda)$.

If the domain wall is thin, $\pi\delta/(4\lambda) \ll 1$, Eq. (10) yields

$$\frac{J_y(x)}{cM} = \frac{d_F}{2\lambda}\left[\tanh \frac{x}{\delta} - \frac{2}{\pi} \operatorname{Im} \psi\left(\frac{1}{4} + i\frac{x}{2\pi\delta}\right) + \frac{\delta}{2\lambda} g(x)\right], \tag{11}$$

where

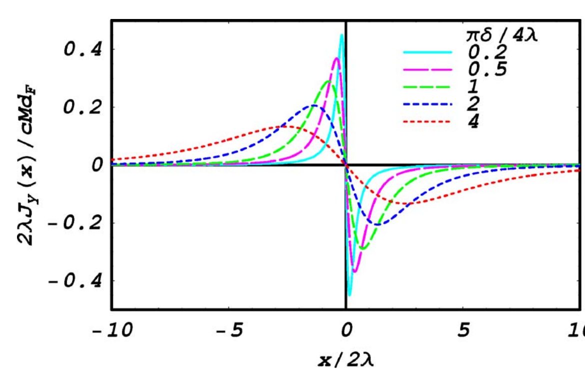

FIG. 4. (Color online) The plot of $2\lambda J_y(x)/(cMd_F)$ as a function of $x/(2\lambda)$ in the case of the thin SF bilayer, $d_S, d_F \ll \lambda_L$. The parameter $\pi\delta/(4\lambda) = 0.2, 0.5, 1, 2, 4$ (from top to bottom in the left panel).

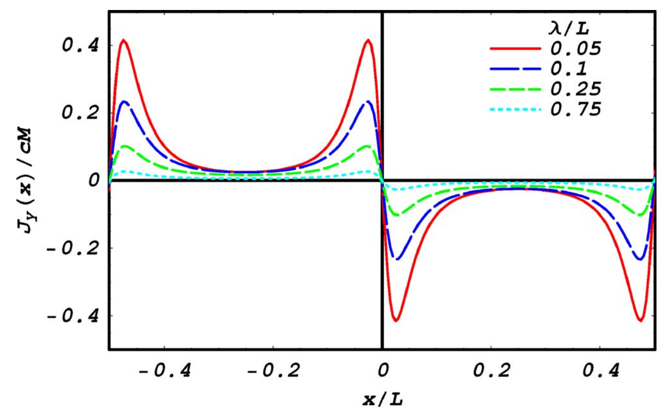

FIG. 5. (Color online) The plot of $J_y(x)/(cM)$ as a function of x/L in the case of the thin SF bilayer, $d_S, d_F \ll \lambda_L$. We use $\delta/L = 0.02$, $d_F/L = 0.1$ and parameter $\lambda/L = 0.05, 0.1, 0.25, 0.75$ (from top to bottom in the left panel).

$$g(x) = \begin{cases} \frac{\pi}{2} \operatorname{sign} x \cos \frac{x}{2\lambda} + f\left(\frac{x}{2\lambda}\right), & |x| \gg \delta, \\ \frac{x}{\delta}, & |x| \ll \delta. \end{cases} \tag{12}$$

Here $f(x) = \sin x \operatorname{ci}(x) - \cos x \operatorname{si}(x)$ with $\cos(x)$ and $\operatorname{sign}(x)$ being the cosine and sine integral functions. In the opposite case $\pi\delta/(4\lambda) \gg 1$ the current distribution $J_y(x)$ is given as

$$\frac{J_y(x)}{cM} = -\frac{d_F}{\delta} \frac{\tanh x/\delta}{\cosh x/\delta}. \tag{13}$$

Equations (11) and (13) prove that $J_y(x)$ increases linearly with x for $x \ll \delta$ and decreases algebraically for large x. The position of the maximum of the absolute value $|J_y(x)|$ is situated at $L_s \propto \delta$ and the value at the maximum $|J_y^m| \propto cM(d_F/\delta)\min\{1, \delta/\lambda\}$. As one can see, therefore (Fig. 4), the current distribution for the thin SF bilayer is qualitatively different from the one for the thick SF bilayer (Fig. 2).

In the case of *multi* domain wall structure in the F layer we have performed evaluations of the sum in Eq. (9) numerically and have obtained the results for the current distribution $J_y(x)$ presented in Fig. 5. We mention that the two maximum structures survive even for λ of the order of L for the thin SF bilayer.

It is known that the lower critical field for the thin S film is much smaller than for the bulk superconductor. Therefore, it is possible that even small magnetization collects at domain walls can induce a vortex in the thin S film.[14] Let us assume that there is a single vortex in the thin S film situated at $x=0$. The magnetic field becomes a sum of the magnetic field induced by the domain walls and the magnetic field of the vortex. The free energy can be written as

$$F = \int d^3\mathbf{r}\left(\frac{\mathbf{B}^2}{8\pi} + \frac{\lambda^2}{8\pi}|\nabla \times \mathbf{B}|^2 - \mathbf{MB}\right), \tag{14}$$

where $\mathbf{B}$ denotes the total magnetic field. The difference $\mathcal{F}$ of the free energy for the state with the vortex and the free energy for the state without the vortex is given as follows:[23]

$$\mathcal{F}=\frac{\phi_0}{4\pi}H_{c_1}\lambda\left(1-\frac{M}{M_c}\right), \quad M_c=\frac{H_{c_1}}{4\pi}\frac{\lambda}{d_F}\mathcal{G}(\delta,\lambda,L), \quad (15)$$

where the flux quantum $\phi_0=ch/(2e)$ with h being the Plank constant and e the electron charge. The $H_{c_1}=(\phi_0/4\pi\lambda^2)\ln\lambda/\xi$ is the lower critical field in the thin S film without the F film and

$$\mathcal{G}(\delta,\lambda,L)=\left(\frac{2\pi}{L}\sum_{n=0}^{\infty}\frac{1}{\cosh\frac{\pi q_n\delta}{2}}\frac{\delta}{(1+2q_n\lambda)^2}\right)^{-1}. \quad (16)$$

The $\mathcal{F}$ becomes negative if $M>M_c$ and vortices can proliferate into the S film until vortex-vortex interaction stops it or, that is more probable, the domain wall changes to Néel domain-wall-type to reduce the free energy. In the most interesting case of a single domain wall we find

$$M_c=\frac{H_{c_1}}{4\pi}\frac{\lambda}{d_F}\begin{cases}2\lambda/\delta, & \pi\delta/4\lambda\ll 1,\\ 1-32G\lambda/(\pi^2\delta), & \pi\delta/4\lambda\gg 1,\end{cases} \quad (17)$$

where $G\approx 0.916$ stands for the Catalan constant. We notice[26] that M_c for the case of thick F film, $d_F\gg\lambda_L\gg d_S$ has been found in Ref. 13.

Our main task was to describe transport properties of SF structures where the ferromagnet and the superconductor are coupled through a thick insulator layer (the proximity effect is weak). Below we outline what would happen with transport properties of dirty[9] SF structures due to proximity effects. We assume again that there is no rearrangement of the domain configuration due to the superconductor. When $d_S\gg\xi$ the proximity effect in the superconductor develops more or less as penetration of the magnetization into the superconductor over a distance of the order of ξ.[27] For $d_S\lesssim\xi$ superconducting critical temperature, T_c of the bilayer can be suppressed by the proximity of the F layer, see, e.g., Ref. 9. Meanwhile, magnetization of Bloch domain walls induces in S layers below the bilayer superconducting critical temperature T_c a supercurrent that qualitatively behaves as it was described in the first part of this paper. Consider the case $d_S\ll\xi$, $\xi_F=\sqrt{D_F/E_{\text{ex}}}\ll\xi$ and $E_{\text{ex}}\tau_F\ll 1$, where D_F is the ferromagnet diffusion coefficient and τ_F the mean free path in F. Then the bilayer can be described within the framework of Usadel equations[28,29] and it can be viewed as a "ferromagnetic superconductor" with effective parameters:[29] the superconducting gap Δ_{eff}, the exchange field $\mathbf{E}_{\text{ex}}^{(\text{eff})}$ (here notations of Ref. 29 are used). It is also known that superconductivity survives in this system (if the exchange field in F is homogeneous) until $\mathbf{E}_{\text{ex}}^{(\text{eff})}<\Delta_{\text{eff}}^{(0)}$, where $\Delta_{\text{eff}}^{(0)}$ is the gap at $\mathbf{E}_{\text{ex}}^{(\text{eff})}=0$.[29] Domain wall structures of the ferromagnet make the effective exchange field nonhomogeneous. Solving Usadel equations in this thin bilayer with the help of the method described in Ref. 9 we find that if $\mathbf{E}_{\text{ex}}^{(\text{eff})}(\mathbf{r})$ changes its direction on scales of the order of ξ or smaller (this is nearly so in certain samples used in experiments Ref. 1), then the superconductivity in the bilayer may survive at larger amplitudes at the effective exchange field than in the homogeneous case: if $\sqrt{\langle(\mathbf{E}_{\text{ex}}^{(\text{eff})})^2\rangle}>\Delta_{\text{eff}}^{(0)}$, where $\langle(\mathbf{E}_{\text{ex}}^{(\text{eff})})^2\rangle=\int d\mathbf{r}[\mathbf{E}_{\text{ex}}^{(\text{eff})}(\mathbf{r})]^2/V$, V is the bilayer volume.[23]

In conclusion, domain wall effects in ferromagnet-superconductor structures are investigated. We find the distribution of current in the superconductor induced by magnetic field of Bloch domain walls, calculate the "lower critical" magnetization of the ferromagnet that provides vortices in the superconductor.

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